Name: ____________________

MA 1118 - Multivariable Calculus
Quiz 3 - Quarter I - AY 02-03

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. No notes or tables permitted.

---

1. (10 points) Consider the parametric curve given by

$$\left.\begin{aligned} x &= 1 + e^{2t} \\ y &= t^2 + 3t \end{aligned}\right\} \quad , \quad -1 < t < 2$$

a. Find the slope of the tangent line to the curve at $t = 0$.

solution:

We know that, in general,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+3}{2e^{2t}}$$

represents the slope of the tangent to the curve at t. Therefore, at $t = 0$, the slope of the tangent is:

$$\frac{2(0)+3}{2e^{2(0)}} = \frac{3}{2}$$

b. Set up, but **do not evaluate** an integral that represents the (arc) length of this curve.

solution:

We know that, in general, the arc length between t_0 and t_1 is given by

$$\int_{t_0}^{t_1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Therefore, in this instance, the arc length between $t = -1$ and $t = 2$:

$$\int_{-1}^{2} \sqrt{\left(2e^{2t}\right)^2 + (2t+3)^2}\, dt$$

2. (10 points) Convert the following polar equation to Cartesian form and sketch the resulting curve:

$$r \;=\; 2\cos(\theta) \;-\; 4\sin(\theta)$$

solution:

In this case we can use the facts that

$$r = \sqrt{x^2+y^2} \quad , \quad \cos(\theta) = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}} \quad \text{and} \quad \sin(\theta) = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

to rewrite the equation as:

$$\sqrt{x^2+y^2} = 2\,\frac{x}{\sqrt{x^2+y^2}} \;-\; 4\,\frac{y}{\sqrt{x^2+y^2}}$$

or $\quad x^2+y^2 = 2x-4y \quad \Longrightarrow \quad x^2-2x+y^2+4y=0$

Completing the square yields

$$(x-1)^2+(y+2)^2 = 5$$

which can easily seen to be a circle of radius $\sqrt{5}$, centered at $(1,-2)$, i.e.: